LOYOLA COLLEGE (AUTONOMOUS) CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **NOVEMBER 2024**

PMT1MC01 – LINEAR ALGEBRA

Date: 08-11-2024 Dept. No. [ ] Max. : 100 Marks

Time: 01:00 pm-04:00 pm

	SECTION A – K1 (CO1)
	Answer ALL the questions **(5 x 1 = 5)**
1	**Answer the following**
a)	What is a minimal polynomial.
b)	Define projection operator.
c)	Let V be a vector space over F and T be an operator on V. Show that $T-$¿ annihilator of α divides the minimal polynomial for T.
d)	Give an example for an nilpotent matrix of order 3.
e)	Write an Inner product on R^3.
	SECTION A – K2 (CO1)
	Answer ALL the questions **(5 x 1 = 5)**
2	**MCQ**
a)	The characteristic values of T are 0 and 1. Then the characteristic values of T^2+2T are i)1,2 ii) 0, 3 iii) 4, -1 iv) $-1,-2$
b)	Degree of the minimal polynomial of a matrix $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ are i) 1 ii) 0 iii) 2 iv) 4
c)	If V has a basis containing characteristic vectors of T, then T is i) not diagonalizable ii) diagonalizable iii) nilpotent iv) zero operator
d)	For a companion matrix characteristic polynomial i) is equal to minimal polynomial ii) is a projection iii) has even degree iv) is not equal to minimal polynomial
e)	For $a, b \in R$, let $p(x,y)=a^2 x_1 y_1+ab x_2 y_1+ab x_1 y_2+b^2 x_2 y_2$, $x=(x_1,x_2)$, $y=(y_1,y_2) \in R^2$. For what values of a and b does $p: R^2 \times R^2 \rightarrow R$ define an inner product i) $a>0, b>0$ ii) $ab>0$ iii) $a=0, b=0$ iv) For no values of a, b
	SECTION B – K3 (CO2)
	Answer any THREE of the following **(3 x 10 = 30)**
3	Write about input - output economic models.

4	Find projection E which projects R^2 onto the subspace spanned by $(1,-1)$ along the subspace spanned by $(1,2)$.
5	Let T be a linear operator on R^2 which has matrix in the standard basis $\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$ and W_1 be subspace spanned by $(1,0)$. Prove that there is no invariant subspace W_2 under T satisfying $R^2 = ¿\, W_1 \oplus W_2$
6	Let α be any non-zero vector in V and let $p_{\square}$ be the T-annihilator of *α*. Let *Z(α; T)* be the cyclic subspace generated by *α*. Discuss about the relations between $p_{\square}$ and *Z(α; T)*
7	Let $(¿)$ be the standard inner product on R^2. $\alpha=(1,2), \beta=(-1,1)$. If γ is a vector such that $(\alpha / \gamma)=-1$ and $(\beta / \gamma)=-1$ then find γ.
	SECTION C – K4 (CO3)
	Answer any TWO of the following **(2 x 12.5 = 25)**
8	Let T be a linear operator on a finite-dimensional space V. Let $c_1, \ldots, c_k$ be the distinct characteristic values of T and let W_i be the null space of $(T - c_iI)$. The following are equivalent. (i) T is diagonalizable. (ii) The characteristic polynomial for T is $f = (x - c_1)^{d1} \ldots (x - c_k)^{dk}$ and dim $W_i = d_i$, i =1, … k. (iii) dim $W_1 + \ldots +$dim $W_k =$ dim V
9	Let T be a linear operator on the finite-dimensional vector space V over the field F. Suppose that the minimal polynomial for T decomposes over F into a product of linear polynomials. Then prove that there is a diagonalizable operator D on V and a nilpotent operator N on V such that (i) T = D + N ; (ii) DN=ND Also prove that the diagonalizable operator D and the nilpotent operator N are uniquely determined by (i) and (ii) and each of them is a polynomial in T.
10	Write about companion matrix .
11	Let W be a finite dimensional subspace of an inner product space V and let E be the projection of V on W. Prove that E is a idempotent linear transformation of V onto W. Also prove that $V = W \oplus W^{\perp}$
	SECTION D – K5 (CO4)
	Answer any ONE of the following **(1 x 15 = 15)**
12	Diagonalize the matrix $A=\begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$. Also write the minimal polynomial for A
13	Discuss about the applications of nilpotent matrices and primary decomposition in the construction of Jordon form with an illustration.
	SECTION E – K6 (CO5)
	Answer any ONE of the following **(1 x 20 = 20)**
1	Decompose a finite dimensional vector space into cyclic subspaces.

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15	For any linear operator T on a finite-dimensional inner product space V, show that there exists a unique linear operator T* on V such that $(T\alpha\|\beta) = (\alpha\|T^*\beta)$ for all α, β in V. Also discuss about the properties of an adjoint operator with an example.

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